Hans Duistermaat: the Man and his Mathematics

Johan Kolk

Mathematical Institute
Utrecht University

Staff Colloquium UvA – April 2011

Thanks to Erik van den Ban
Jan Brandts
Wilberd van der Kallen
Job Kuit
V.S. Varadarajan (UCLA)

Outline

Johannes Jisse Duistermaat

- Biographical Data
- Photographs
- Chess
- Concentration and Persistence
- Impact in Mathematics
- Video

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Biographical Data
Photographs
Chess
Concentration and Persistence
Impact in Mathematics
Video
Mathematics
Thesis
Proof of FIT = Fourier Inversion Theorem
Fourier Integral Operators
Singularities and Eigenvalues
Semisimple Lie Groups
Duistermaat–Heckman Formula
Classical Mechanics
Books
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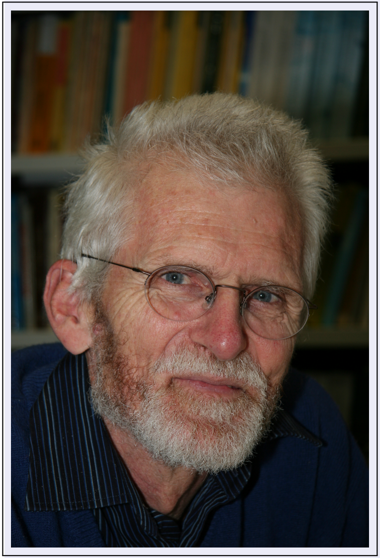
Epilogue

Biographical Data

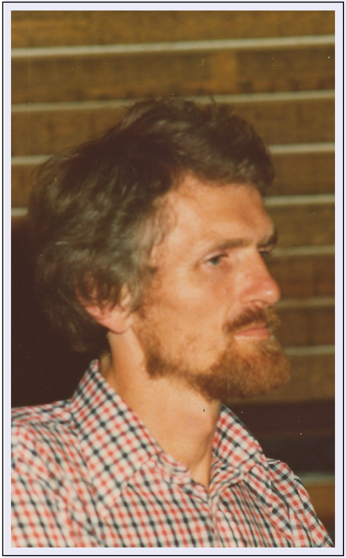
20-12-1942	Born in The Hague
1947-57	In the Netherlands Indies (= Indonesia)
1968	Thesis at UU (= Utrecht Univ.)
1969-70	In Lund, Sweden, funded by Shell Prize
1971	Catholic Univ. Nijmegen (= Radboud Univ.)
1972	Full professor Nijmegen
1974	Professor Pure and Applied Mathematics UU
1982	Member KNAW (= Royal Netherlands Academy of Arts and Sciences)
2005-09	KNAW professor. “One of the world’s most important mathematicians in differential equations”
19-03-2010	Passed away in Utrecht

Photographs



2006
Hans as we recall him from recent years.

Photographs



1977
The party following the defense of my thesis.

Chess

Hans was an accomplished chess player.
In a simultaneous match of world chess champion A. Karpov playing ten opponents in 1977, Hans was the only one to score a draw. He missed an opportunity to win.

Concentration and Persistence

Everyone who met Hans, had the impression of a very powerful mind, very ascetic in temperament, extraordinarily direct and honest.
His powers of concentration and persistence were immense: in mathematics, discussions, sports, ...

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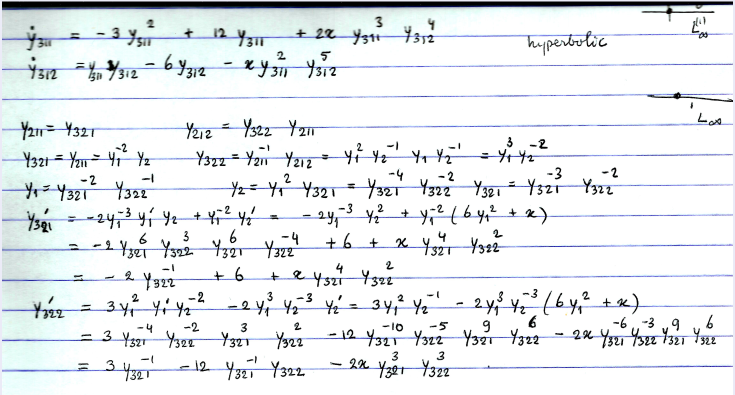
For instance, when Hans wanted to go wind-surfing on the Pacific near Los Angeles. Floor, my wife took him there by car. Hans started out at 2 pm. in the Marina del Rey, an extensive harbor for yachts, and promised to be back in about 2 hours. At 5:30 Floor called me in panic, and Hans returned enthusiastically at 6:30: “it took a while to reach the ocean, but I had a wonderful time.”

Concentration and Persistence

Fragment of computation for a paper, to appear in Nonlinearity.



$$\dot y_{311} = -3\, y_{311}^{2} + 12\, y_{311} + 2x\, y_{311}^{3}\, y_{312}^{4}$$

hyperbolic $L_\infty^{(1)}$

$$\dot y_{312} = y_{311}\, y_{312} - 6\, y_{312} - x\, y_{311}^{2}\, y_{312}^{5}$$

L_∞

$$y_{211} = y_{321} \qquad y_{212} = y_{322}\, y_{211}$$

$$y_{321} = y_{211} = y_1^{-2}\, y_2 \qquad y_{322} = y_{211}^{-1}\, y_{212} = y_1^{2}\, y_2^{-1}\, y_1\, y_2^{-1} = y_1^{3}\, y_2^{-2}$$

$$y_1 = y_{321}^{-2}\, y_{322}^{-1} \qquad y_2 = y_1^{2}\, y_{321} = y_{321}^{-4}\, y_{322}^{-2}\, y_{321} = y_{321}^{-3}\, y_{322}^{-2}$$

$$y_{321}' = -2 y_1^{-3}\, y_1'\, y_2 + y_1^{-2}\, y_2' = -2 y_1^{-3}\, y_2^{2} + y_1^{-2}(6 y_1^{2} + x)$$

$$= -2\, y_{321}^{6}\, y_{322}^{3}\, y_{321}^{6}\, y_{322}^{-4} + 6 + x\, y_{321}^{4}\, y_{322}^{2}$$

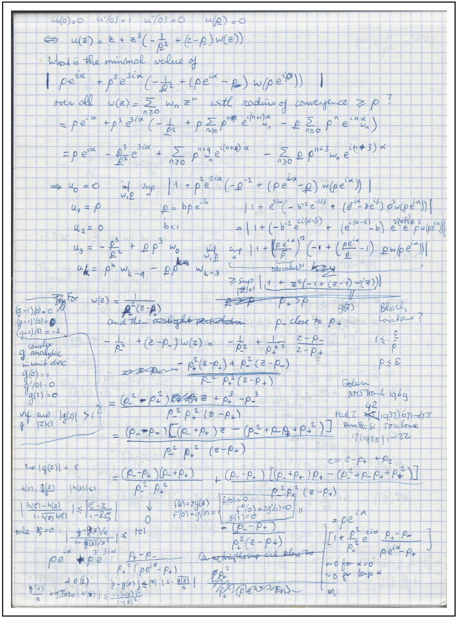
$$= -2\, y_{322}^{-1} + 6 + x\, y_{321}^{4}\, y_{322}^{2}$$

$$y_{322}' = 3 y_1^{2}\, y_1'\, y_2^{-2} - 2 y_1^{3}\, y_2^{-3}\, y_2' = 3 y_1^{2}\, y_2^{-1} - 2 y_1^{3}\, y_2^{-3}(6 y_1^{2} + x)$$

$$= 3\, y_{321}^{-4}\, y_{322}^{-2}\, y_{321}^{3}\, y_{322}^{2} - 12\, y_{321}^{-10}\, y_{322}^{-5}\, y_{321}^{9}\, y_{322}^{6} - 2x\, y_{321}^{-6}\, y_{322}^{-3}\, y_{321}^{9}\, y_{322}^{6}$$

$$= 3\, y_{321}^{-1} - 12\, y_{321}^{-1}\, y_{322} - 2x\, y_{321}^{3}\, y_{322}^{3}$$

Concentration and Persistence



Impact in Mathematics

Brilliant research mathematician in geometric analysis.

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Active in different parts of math

- Ordinary and partial differential equations
- Discrete integrable systems
  (difference equations, complete sets of conservation laws, exact solutions)
- Classical mechanics
- Analysis on (semisimple) Lie groups
  (= smooth manifolds that are (special) groups.
  Example: certain groups of matrices)
- Symplectic differential geometry
  (= differential geometry describing the phase spaces of classical mechanical systems and generalizations)
- Algebraic geometry

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- 23 PhD students and furthermore post-docs
- International contacts, in particular, USA, France, Indonesia
- Inspiring teacher

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Author citations in Math Reviews Citation Database
1071 Duistermaat

For comparison
2079 A. Schrijver
1594 N.Yu. Reshetikhin
7618 J.-P. Serre (Fields medalist)
667 J.-C. Yoccoz (Fields medalist)

Video

Rothschild Visiting Professor
Discrete Integrable Systems programme
Isaac Newton Institute for Mathematical Sciences
Cambridge, U.K.
18-05-2009

`http://www.sms.cam.ac.uk/media/603139`

Thesis

Energy and entropy as real morphisms for addition and order.
Foundations of thermodynamics.
Originated in seminar conducted by Günther K. Braun, professor of applied mathematics, and a chemist.

Braun died tragically one year before defense of thesis.
Thesis advisor: Hans Freudenthal, geometer.

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Thesis lead to dissent between physicists and mathematicians at UU.
Hans dropped the topic.
Nevertheless an important influence in his career.

Thesis

Studied works of Sophus Lie on contact and symplectic transformations (modern terminology).

Lars Hörmander, analyst and Fields medalist, used Hans' knowledge in formulation of calculus of
FIO's = Fourier integral operators.
This class of operators contains partial differential operators as well as classical integral operators as special cases,
while composition of these operators is often well-defined.

One goal is to obtain inverses of differential operators.
Useful in the theory of high-frequency waves and propagation of singularities, like shock waves.

Proof of FIT = Fourier Inversion Theorem

Fourier transform $\mathcal{F}$ acting on Schwartz function ϕ on $\mathbf{R}^n$ (= smooth and rapidly decaying at infinity)

$$\mathcal{F}\phi(\xi) = \int_{\mathbf{R}^n} e^{-i\langle x,\xi\rangle}\phi(x)\,dx = \int_{\mathbf{R}^n} e_{-i\,\xi}\,\phi\,dx.$$

$\mathcal{F}\phi$ is Schwartz too. FIT = Fourier Inversion Theorem FIO

$$\phi(x) = \frac{1}{(2\pi)^n}\int_{\mathbf{R}^n} e^{i\langle x,\xi\rangle}\mathcal{F}\phi(\xi)\,d\xi.$$

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$$\phi(x) = \frac{1}{(2\pi)^n}\int_{\mathbf{R}^n} e^{i\langle x,\xi\rangle}\mathcal{F}\phi(\xi)\,d\xi.$$

Sufficient to prove for $x = 0$:

$$(2\pi)^n\,\phi(0) = \int_{\mathbf{R}^n}\mathcal{F}\phi(\xi)\,d\xi.$$

Indeed, integration by parts leads to local and global intertwining identities, for $\partial_j := \partial/\partial x_j$, 1L

$$\mathcal{F}\circ\partial_j = i\,\xi_j\circ\mathcal{F}, \qquad \mathcal{F}\circ {T_a}^* = e_{i\,a}\circ\mathcal{F}, \quad {T_a}^*\phi(x) = \phi(x+a).$$

For general case of FIT, replace ϕ by ${T_x}^*\phi$.

Proof of FIT = Fourier Inversion Theorem

Desired: $(2\pi)^n\,\phi(0) = \int_{\mathbf{R}^n} \mathcal{F}\phi(\xi)\,d\xi$. Use Dirac's delta function δ and formally interchange order of integration to rewrite this as

$$(2\pi)^n \int_{\mathbf{R}^n} \delta(x)\,\phi(x)\,dx = \int_{\mathbf{R}^n}\int_{\mathbf{R}^n} e^{-i\langle x,\xi\rangle}\,d\xi\,\phi(x)\,dx.$$

In other words, FIT equivalent to

$$(2\pi)^n\,\delta = \int_{\mathbf{R}^n} e_{-i\,\xi}\,1\,d\xi = \mathcal{F}1.$$

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Tempered distributions are linear functionals acting on Schwartz functions continuously with respect to a suitable topology. Examples: $\delta(\phi) = \phi(0)$, $\psi(\phi) = \int_{\mathbf{R}^n} \psi(x)\,\phi(x)\,dx$. For tempered distribution T, define $(\mathcal{F}T)(\phi) = T(\mathcal{F}\phi)$. Justification: for Schwartz function T we have

$$(\mathcal{F}T)(\phi) = \int_{\mathbf{R}^n} (\mathcal{F}T)(\xi)\,\phi(\xi)\,d\xi = \int_{\mathbf{R}^n} T(\xi)\,\mathcal{F}\phi(\xi)\,d\xi = T(\mathcal{F}\phi).$$

Proof of FIT = Fourier Inversion Theorem

For years discussions on and off about most transparent proof of FIT.
Hans came up with following one-line argument:

$$0 = \mathcal{F}0 = \mathcal{F}(\partial_j 1) = i\,\xi_j\,\mathcal{F}1 \qquad (1 \leq j \leq n).$$

Indeed, follows from local intertwining identity.

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Next, first-order Taylor expansion gives $\phi = \phi(0)\,1 + \sum_j \xi_j\,\psi_j$.
Now $\xi_j\,T = 0$ leads to existence of $c \in \mathbf{C}$ with

$$\mathcal{F}1(\phi) = T(\phi) = \phi(0)\,T1 + \sum_{j=1}^{n} \xi_j\,T(\psi_j) = c\,\delta(\phi).$$

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Testing identity against Gaussian $e^{-\|\cdot\|^2/2}$ gives $c = (2\pi)^n$.
Thus FIT ▸ FIT follows. □

Fourier Integral Operators

For $P(x, \partial)$, linear partial differential operator with variable coefficients, preceding results imply

$$P(x, \partial)\phi(x) \quad = \quad \frac{1}{(2\pi)^n} \int_{\mathbf{R}^n} e^{i\langle x, \xi\rangle} \, P(x, i\xi) \, \mathcal{F}\phi(\xi) \, d\xi$$

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$$
\begin{aligned}
P(x,\partial)\phi(x) &= \frac{1}{(2\pi)^n}\int_{\mathbf{R}^n} e^{i\langle x,\xi\rangle}\, P(x,i\xi)\,\mathcal{F}\phi(\xi)\,d\xi \\
&= \frac{1}{(2\pi)^n}\int_{\mathbf{R}^n\times\mathbf{R}^n} e^{i\langle x-y,\xi\rangle}\, P(x,i\xi)\,\phi(y)\,d(y,\xi)
\end{aligned}
$$

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Relax conditions on phase function χ and on P to get FIO's.

Singularities and Eigenvalues

Influential articles arising from the calculus of FIO's

- (jointly with Hörmander) *FIO's II* (1972)
- *Oscillatory integrals, Lagrange immersions and unfolding of singularities* (1974)
  Agenda for study of singularities of smooth functions and their applications to distribution theory.
  In some sense complementary to FIO's and parallel to work of Vladimir Arnol'd, Dr.h.c. UU
- (jointly with Victor Guillemin) *The spectrum of positive elliptic operators and periodic bicharacteristics* (1975)
  Application of FIO's to the asymptotics of spectra (= distribution of eigenvalues) of elliptic operators.
  Relation between spectra and geometry.
  His most cited paper ▸ SL

These firmly established Hans' mathematical reputation.

Singularities and Eigenvalues

Inventiones math. 29, 39 – 79 (1975)

The Spectrum of Positive Elliptic Operators and Periodic Bicharacteristics

J.J. Duistermaat (Utrecht) and V.W. Guillemin (Cambridge, Mass.)

Introduction

Let X be a compact boundaryless C^∞ manifold and let P be a positive elliptic self-adjoint pseudodifferential operator of order $m>0$ on X. For technical reasons we will assume that P operates on half-densities rather than functions. (We will denote the half-density bundle over X by $\Omega_{\frac{1}{2}}$.) We will also assume that P is a classical pseudodifferential operator in the sense that on every coordinate patch its total symbol $\sigma_P(x,\xi)$ admits an asymptotic expansion

$$\sigma_P(x,\xi) \sim \sum_{j=0}^{\infty} p_{m-j}(x,\xi)$$

with $p_{m-j}(x,\xi)$ homogeneous of degree $m-j$. We recall that the principal symbol p of P is equal to p_m on local coordinates, and the subprincipal symbol is equal to $p_{m-1} - \dfrac{1}{2i} \sum \dfrac{\partial^2 p}{\partial x_j \, \partial \xi_j}$.

Let $\lambda_1, \lambda_2, \ldots$ be the eigenvalues of P. It was remarked by Chazarain in [6] and by ourselves in [11] that the sum $\sum e^{-i \sqrt[m]{\lambda_k}\, t}$ is well-defined as a generalized function of t and that if T is in its singular support then the Hamiltonian vector field

$$H_q = \frac{\partial q}{\partial \xi_j} \frac{\partial}{\partial x_j} - \frac{\partial q}{\partial x_j} \frac{\partial}{\partial \xi_j}, \qquad q = \sqrt[m]{p}$$

Semisimple Lie Groups

Transition to Lie theory was remarkable occurrence.
It demanded complete change of mathematical perspective:
from classical and highly general aspects of PDE's
to very specific questions on semisimple Lie groups.

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It demanded complete change of mathematical perspective:
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to very specific questions on semisimple Lie groups.

Improve error estimates in ▸ G, in special case of compact locally symmetric space X, to prove conjectures of Israel Gel'fand.
Such an X is described in terms of a semisimple Lie group G.

Semisimple Lie Groups

DKV1 (V = V.S. Varadarajan) uses Selberg trace formula, deep generalization of Poisson's summation formula on **R**: ▸ SUM

$$\sum_{\lambda \in \mathbf{Z}} e_{i\,\lambda} = 2\pi \sum_{n \in \mathbf{Z}} \delta_{2\pi\, n}, \qquad \sum_{\lambda \in \mathbf{Z}} \mathcal{F}\phi(\lambda) = 2\pi \sum_{n \in \mathbf{Z}} \phi(2\pi n).$$

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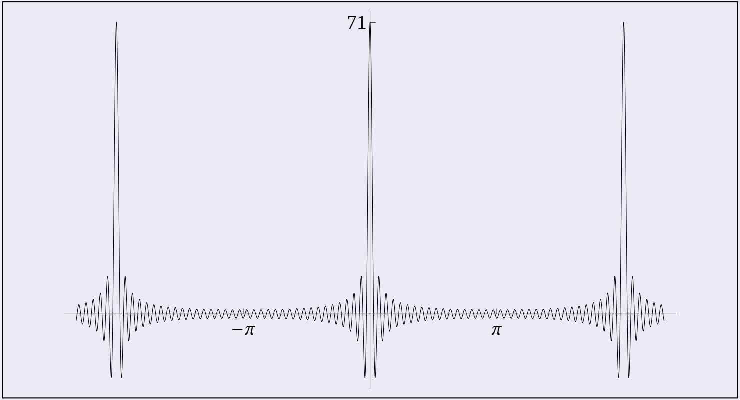
Here $X = \mathbf{R}/2\pi\mathbf{Z}$ and LHS (spectral) in terms of eigenvalues of the Laplace operator $-\partial^2$ on X.
RHS (geometric) in terms of $\mathrm{vol}(X)$ and lengths of periodic geodesics of X.

Simultaneous control of behavior of elementary spherical functions $x \mapsto \phi_\lambda(x)$ on G as both λ and x to infinity independently of each other, by method of stationary phase.

Semisimple Lie Groups

Graph of $\sum_{\lambda=-35}^{35} e_{i\,\lambda}$. ▸ P



Nowhere pointwise convergence.

Semisimple Lie Groups

It was in the second paper (DKV2) that a true synthesis was achieved, of ideas that Hans was truly in love with,
such as oscillatory integrals, caustics, Lagrange immersions,
with the ideas in semisimple Lie theory,
such as orbital integrals and their asymptotics.

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such as oscillatory integrals, caustics, Lagrange immersions,
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such as orbital integrals and their asymptotics.

Highly original suggestion to Van den Ban.
Take the integrals on real flag manifolds that represent elementary spherical functions ϕ_λ, and move them into the complex flag manifolds,
by moving the real cycle into the complex domain.
A generalization of classical technique in complex function theory of deforming the contour,
but now in a geometrically more complex multi-dimensional context.

Duistermaat–Heckman Formula

Gert Heckman wrote thesis under G. van Dijk and Hans on questions concerning irreducible representations of compact Lie groups.
Gert and Hans proved these results could be understood in context of symplectic geometry.

This led to celebrated Duistermaat–Heckman formula
It expresses that Fourier transform of canonical (Liouville) measure on a symplectic manifold is given exactly by stationary phase approximation.

Second most quoted article.
It led to many important generalizations and is frequently used in new applications.
Some years ago famous mathematician J.-M. Bismut declared that he spent most of his life to understand the meaning of this formula.

Classical mechanics

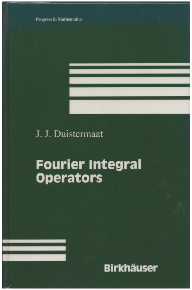
- Periodic solutions of a Hamiltonian system near an equilibrium point
- Monodromy in integrable Hamiltonian systems
- Nonholonomically constrained systems.
  Examples: disk or ball with center of mass not at its geometric center, rolling without slipping on horizontal plane under influence of constant vertical gravitational force.

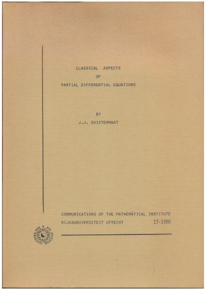
Books

Hans (co-)authored a total of 11 books
(with Richard Cushman, Wiktor Eckhaus, Kolk,
Jędrzej Śniatycki).
3 books were published last year.

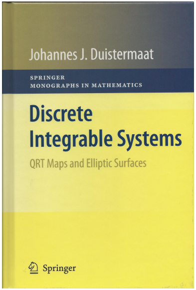
- *Lie Groups*, 2000. Most cited
- *Fourier Integral Operators*, 1973, 1996, 2011.
  Second most cited

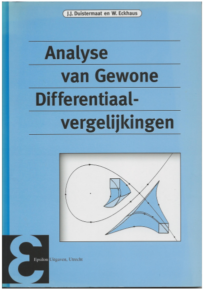
Books


Progress in Mathematics
J. J. Duistermaat
Fourier Integral Operators
Birkhäuser

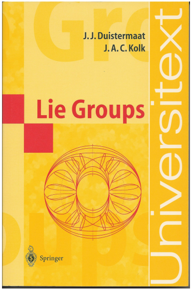

CLASSICAL ASPECTS
OF
PARTIAL DIFFERENTIAL EQUATIONS
BY
J.J. DUISTERMAAT
COMMUNICATIONS OF THE MATHEMATICAL INSTITUTE
RIJKSUNIVERSITEIT UTRECHT
13-1980


Progress in Nonlinear Differential Equations and Their Applications
J.J. Duistermaat
The Heat Kernel Lefschetz Fixed Point Formula for the Spin-c Dirac Operator
Birkhäuser


Johannes J. Duistermaat
SPRINGER MONOGRAPHS IN MATHEMATICS
Discrete Integrable Systems
QRT Maps and Elliptic Surfaces
Springer


J.J. Duistermaat en W. Eckhaus
Analyse van Gewone Differentiaal-vergelijkingen
Epsilon Uitgaven, Utrecht

Books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Lie Groups
Springer
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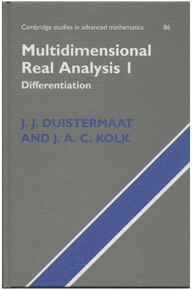
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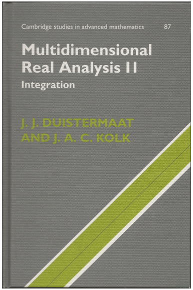
Multidimensional Real Analysis I
Differentiation
J. J. DUISTERMAAT AND J. A. C. KOLK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Multidimensional Real Analysis II
Integration
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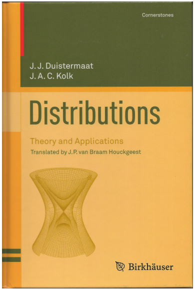
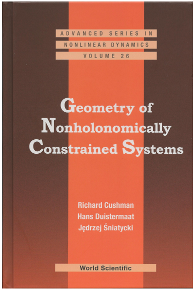
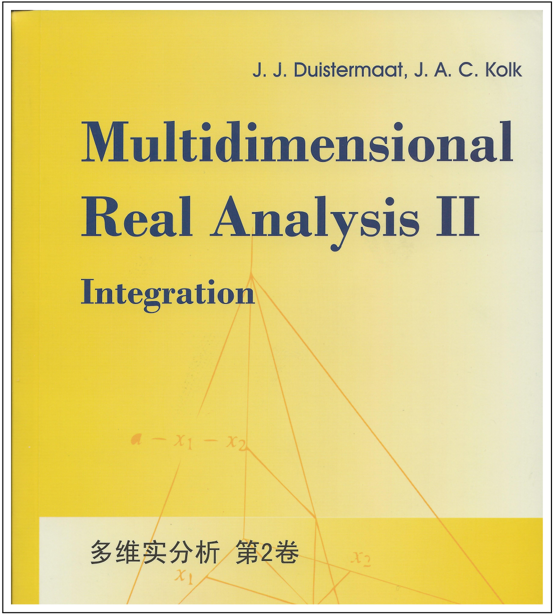

Cornerstones
J. J. Duistermaat
J. A. C. Kolk
Distributions
Theory and Applications
Translated by J. P. van Braam Houckgeest
Birkhäuser


ADVANCED SERIES IN
NONLINEAR DYNAMICS
VOLUME 26
Geometry of Nonholonomically Constrained Systems
Richard Cushman
Hans Duistermaat
Jędrzej Śniatycki
World Scientific

Books



J. J. Duistermaat, J. A. C. Kolk
Multidimensional Real Analysis II
Integration
多维实分析 第2卷

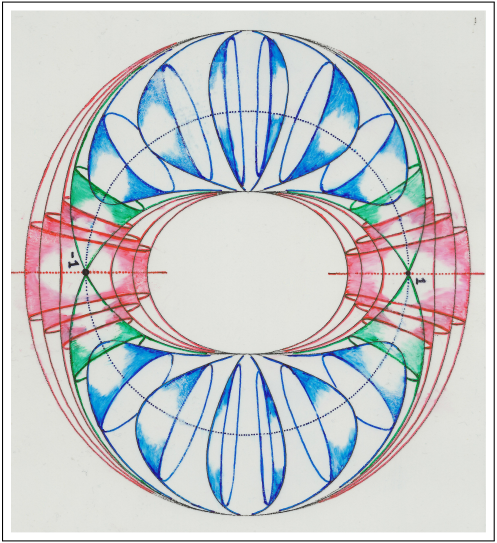
Books

Reviews

- *Fourier Integral Operators*
  Duistermaat's notes have certain features that make them unique
- *The Heat Kernel Lefschetz Fixed Point Formula for the Spin-c Dirac Operator*
  Overall this is a carefully-written, highly readable book on a very beautiful subject
- *Lie Groups*
  A brilliant book on Lie groups
- *Multidimensional Real Analysis*
  The mother of all multivariable calculus books
- *Distributions: Theory and Applications*
  This is a fantastic book.
  …I am ecstatic to have the book by Duistermaat and Kolk in my hands, as it is a gem

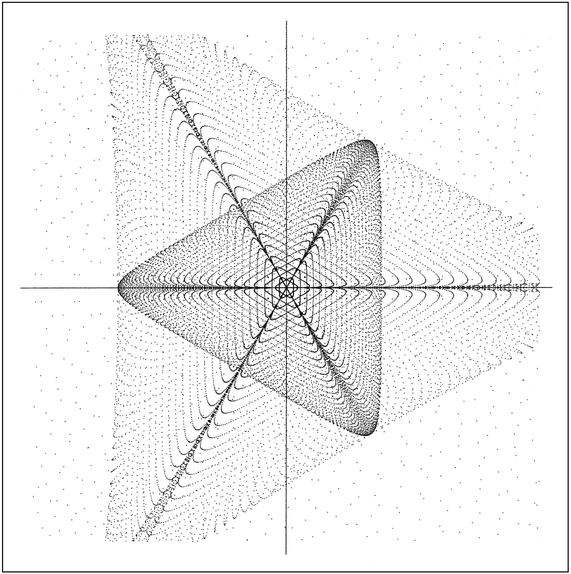
Graphics

Intense interest in correct graphics.
Hand-colored picture of $\mathbf{SL}(2, \mathbf{R})$.



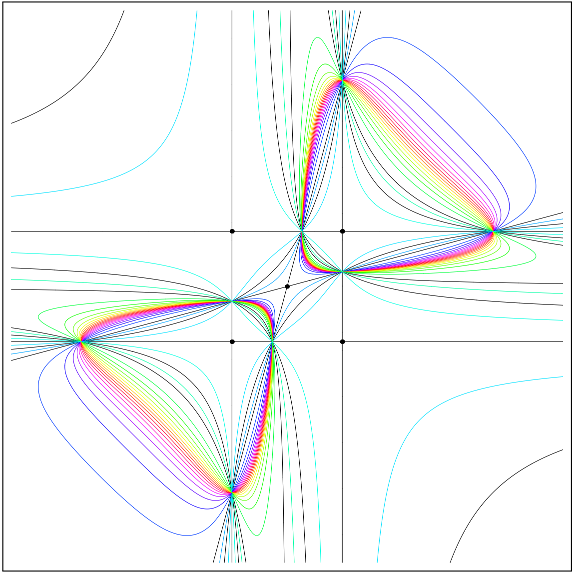
Graphics

Dot for dot on a matrix printer. Image of character of **SU**(3).



Graphics

Pencil of real sine-Gordon curves, with eight real base points.



Epilogue

For much more on Hans' scientific work, consult

- Nieuw Arch. Wiskd. (5) **11** (2010), 235–243
- *Geometric Aspects of Analysis and Mechanics. A Conference in Honor of the 65th Birthday of Hans Duistermaat*, 2011
  to be published by Birkhäuser, Boston
- Notices Amer. Math. Soc., *Obituaries*, to appear
- `http://www.staff.science.uu.nl/~kolk0101/duisinmem.html`

A truly remarkable Renaissance mind

(V.S. Varadarajan)

Epilogue

WONDER, **W**iskunde **onder**zoekschool Nederland
(= The Dutch Research School in Mathematics)
has decided to establish the

Hans Duistermaat Chair for Visiting Professors